Non-Archimedean Analytic Geometry: Theory & Practice

WELCOME BOOKLET

Agence Nationale
de la Recherche


Université de la
Polynésie française

European
Research
Council

Institut
Universitaire
de France

Dear conference participant,

Welcome to French Polynesia.

French Polynesia, or *Polynésie française*, is an overseas country of the French Republic. It comprises about 130 islands partitioned in five archipelagos. The largest and most populous island, Tahiti, is part of the *Îles du Vent* group of the *Îles de la Société* archipelago.

Polynesia has been visited by European explorers since the sixteenth century, with French settlers arriving in the early nineteenth century. France gave French Polynesia the status of an overseas country in 1946; essentially, Polynesians vote in nation-wide French legislative and executive elections, as well as to elect a local government and assembly. Those local bodies have more leeway than those in charge of similar administrative divisions in mainland France, although matters such as justice and defense remain under the authority of the nation.

Tahiti is an extinct volcano, which explains its mountainous center and surrounding lagoon. It is home to 180, 000 people, which comprises about two thirds of the population of French Polynesia. The capital of French Polynesia is the city of Papeete where the conference is held.

Most people in Tahiti speak French, although Tahitian is the mother tongue of many. Other Polynesian languages are widely spoken in the surrounding archipelagos. Common Tahitian words and expressions are also frequently used in French; during your stay you will likely hear most if not all of the following ones:

'ia ora na	hello
maeva	welcome
nana	goodbye
māuruuru	thank you
vahine	woman
tane	man
fenua	the land

Practical Information

Location

The conference will be held at the Muriavai Hall of the Maison de la Culture in downtown Papeete. The participants will stay at the Royal Tahitien hotel located in a tropical garden on the beach of a lagoon.

A free shuttle service will operate from the hotel to the conference center and back according to the following schedule:

	Hotel to conference	*Conference to hotel*
Monday	08:00	18:30
Tuesday	07:40	18:30
Thursday	07:40	21:15
Friday	07:40	18:30

The Royal Tahitien hotel does not provide a complimentary breakfast. However, conference days will begin with a coffee break during which Viennoiseries and fruit juice will be served. Lunches (free of charge for participants) will be held at the Manava restaurant on Monday and Tuesday and at the La Romana restaurant on Thursday and Friday. Both restaurants are a ten-minutes walk from the Maison de la Culture.

Events

On Wednesday an excursion will take us to the inner parts of the island up the Papenoo valley. We will have lunch at the Relais de la Maroto with a wonderful view of the surrounding mountains. The departure is scheduled at 08:15 from the Royal Tahitien hotel, we should be back by 18:00. Please dress comfortably for this trip.

Later on the same day, the shuttle will pick us up at the Royal Tahitien at 18:40 and drive us to the conference dinner held at the InterContinental Hotel. This will be an occasion to discover the Marquesas Islands dances performed by a group of warriors. The buffet will feature Polynesian and Marquesas specialties. The shuttle bus back to the Royal Tahitien will depart from the InterContinental hotel at 21:30.

If you come as a couple and your partner wishes to participate to the excursion and/or the conference dinner, please let us know about it early on Monday so that we are able to make necessary arrangements. Note that this is will imply additional charges.

On Thursday evening we plan to dine together at the roulottes of the Vaiete square and have a beer at “Les 3 Brasseurs”.

	Monday August 24	**Tuesday** August 25
8:00		*coffee*
8:30	*coffee & registration*	Florent Martin
9:00	*opening remarks*	
9:30	Laura DeMarco	*coffee*
10:00		Emmanuel Lepage
10:30	Juan Rivera-Letelier	
11:00		Jérôme Poineau
11:30		
12:00		
12:30	*lunch*	*lunch*
13:00		
13:30		
14:00		
14:30	Francesco Baldassarri	Kazuhiro Fujiwara
15:00		
15:30	*coffee*	*coffee*
16:00	Ruochuan Liu	Fumiharu Kato
16:30		
17:00	Ambrus Pal	Marc Chapuis
17:30		

Wednesday August 26	**Thursday** August 27	**Friday** August 28
	coffee	*coffee*
	Antoine Ducros	Klaus Künnemann
	coffee	*coffee*
	Joe Rabinoff	Omid Amini
	Chengyang Xu	Kazuhiko Yamaki
free	*lunch*	*lunch*
	Michael Temkin	Bertrand Rémy
	coffee	*coffee*
	Shou-Wu Zhang	Vladimir Berkovich
	Kentaro Mitsui	

Talk Abstracts

Monday 09:30, Laura DeMarco

Variation of canonical height, illustrated

Around 1990, Joe Silverman wrote a series of three articles on the variation of canonical height in families of elliptic curves. I will discuss connections between these results and dynamical systems on $\mathbb{P}^1$ (and an associated Berkovich space). As the height functions define dynamical "bifurcation measures" on the base variety, I will show illustrations of these measure densities. In new work with Dragos Ghioca, we exploit these ideas to study rationality of canonical heights for maps of $\mathbb{P}^1$.

Monday 10:30, Juan Rivera-Letelier

On the equidistribution of points of small toric height

On a toric variety, we study the asymptotic distribution of points that are small with respect to a given toric height. In this setting, the usual method to prove equidistribution, introduced by Ullmo, Szpiro, Zhang, can only be applied to Weil heights. In fact, this method requires the height function to satisfy Zhang's inequality with an equality, and recently Burgos, Philippon, and Sombra showed that this only holds for Weil heights. Nevertheless, we show that for smooth toric heights the equidistribution of points of small height does hold, and as a consequence we derive the Bogomolov property for these heights. We also give a host of examples of non-smooth heights for which equidistribution fails. We also give the first example of a height for which the Bogomolov property fails.

This is joint work with Burgos, Philippon, and Sombra.

Monday 14:30, Francesco Baldassarri

A p-adically entire function with integral values on $\mathbb{Q}_p$ and additive characters of perfectoid fields

We give an essentially self-contained proof of the fact that a certain p-adic power series

$$\Psi = \Psi_p(T) \in T + T^{p-1}\mathbb{Z}[[T^{p-1}]],$$

which trivializes the addition law of the formal group of Witt p-covectors is p-adically entire and assumes values in $\mathbb{Z}_p$ all over $\mathbb{Q}_p$. We also carefully examine its valuation and Newton polygons. For any perfectoid field extension $(K, \|)$ of $(\mathbb{Q}_p, \|_p)$ contained in $(\mathbb{C}_p, \|_p)$, and any pseudo-uniformizer $\varpi = \left(\varpi^{(i)}\right)_{i\geq 0}$ of $K^\flat$, we consider the element

$$\pi = \pi(\varpi) := \sum_{i\geq 0} \varpi^{(i)} p^i + \sum_{i<0} \left(\varpi^{(0)}\right)^{p^{-i}} p^i \in K.$$

We use the isomorphism between the Witt and the Cartier (hyperexponential) group over $\mathbb{Z}_{(p)}$, which we extend to their p-divisible closures, and the properties of Ψ_p, to show that the map $x \mapsto \exp(\pi x)$, a priori only defined for $v_p(x) > \frac{1}{p-1} - v_p(\pi)$, extends to a continuous additive character

$$\Psi_\varpi : \mathbb{Q}_p \to 1 + K^{\circ\circ}.$$

A similar character for the cyclotomic p-extension of $\mathbb{Q}_p$ appears in Colmez' work. I will also give the numerical computation of the first coefficients of Ψ_p, for small p.

Monday 16:00, Ruochuan Liu

Finiteness of cohomology of relative (ϕ, Γ)-modules

We will show finiteness of the higher direct images of geometric (ϕ, Γ)-modules for proper smooth morphisms of smooth rigid analytic varieties. As a consequence, for proper smooth rigid analytic varieties over finite extensions of $\mathbb{Q}_p$, the proetale cohomology groups of $\mathbb{Q}_p$-local systems are all finite dimensional $\mathbb{Q}_p$-vector spaces.

This is joint work with Kedlaya.

Monday 17:00, Ambrus Pal

Rigid cohomology over Laurent series fields

It is possible to refine Berthelot's rigid cohomology over Laurent series fields of characteristic p to a cohomology theory taking values in (ϕ, ∇)-modules over the bounded Robba ring, not just over the larger Amice ring, asfurnished by Berthelot's original theory. One gets the refined theory by replacing Berthelot's notion of a rigid frame with a suitably modified version, which however requires the usage of adic spaces instead of rigid analytic spaces. Nevertheless the arguments of the subject all carry over without much difficulty to prove the finiteness for this cohomology theory, and a comparison theorem with Berthelot's original construction. As a consequence we can attach Weil–Deligne representations to the rigid cohomology of varieties over Laurent series fields using Marmora's functor, and hence complete existing results and conjectures on independence and monodromy-weight. This is joint work with Chris Lazda.

Tuesday 08:30, Florent Martin

Two connectedness results on Berkovich spaces

We prove two topological results for Berkovich k-analytic spaces. First, when k is discretely valued, we give a new proof and generalize a result of Siegfried Bosch about the connectedness of tubes over a closed point in the reduction of a strictly k-affinoid space. Secondly, we prove that a subanalytic set has finitely many connected components, and when k is discretely valued, we show that each connected component is itself subanalytic. We use as a main tool a result of A.J. de Jong relating special formal schemes and bounded functions on their generic fibers.

Tuesday 10:00, Emmanuel Lepage

Tempered fundamental group of curves

The tempered fundamental group of a non-archimedean analytic curves classifies a category of covers encompassing at the same time finite étale covers and topological covers of the Berkovich analytic space. In this talk, we will discuss what can be recovered of a curve over an algebraically closed non-archimedean field of mixed characteristics from its tempered fundamental group.

Tuesday 11:00, Jérôme Poineau

Theorems A and B for Berkovich spaces over $\mathbb{Z}$

Although Berkovich spaces usually appear in a non-archimedean setting, their general definition actually allows arbitrary Banach rings as base rings, e.g. $\mathbb{Z}$ endowed with the usual absolute value. Over the latter, Berkovich spaces look like fibrations that contain complex analytic spaces as well as p-adic analytic spaces for every prime number p. After recalling the main known properties of those spaces, we will show that relative disks of arbitrary dimension satisfy the conclusions of Cartan's theorems A (global generation) and B (vanishing of higher cohomology) for coherent sheaves. Those results can be considered as counterparts of the foundational theorems of Kiehl and Tate in the usual p-adic setting.

Tuesday 14:30, Kazuhiro Fujiwara

Proper dominant descent in rigid geometry

In this talk some basic and foundational aspects of rigid geometry is discussed. If one starts with formal schemes, to make the functoriality work, it is necessary to introduce non-noetherian adic rings in the theory. I will show that there is a nice class of adic rings for this purpose. Moreover I hope to discuss the expected global properties, in particular proper dominant descent.

Tuesday 16:00, Fumiharu Kato

Zariski Main Theorem for henselian rigid space

This is a joint-work with Shuji Saito (Tokyo Institute of Technology). We announce an analogue of Zariski Main Theorem in henselian rigid geometry, describing the sketch of the proof, and discuss a few possible applications in algebraic geometry.

Tuesday 17:00, Marc Chapuis

Tamely ramified forms of discs and annuli

Using Michael Temkin's theory of graduated reduction, Antoine Ducros has shown that any tamely ramified form of an open polydisc is trivial. Later, Tobias Schmidt proved that the result holds for closed discs. In fact we have a similar statement both for polydiscs, either closed or open, and for (one-dimensional) annuli, either closed or open in one or the other of their edges. We shall recall the main steps of Ducros' proof which we follow and show broadly how they can be adapted to the difficulties that appear when one substitutes closed to open discs, or annuli to discs.

Thursday 08:30, Antoine Ducros

Reified valuation with infinitesimal elements and skeleta of Berkovich spaces

Let S_n be the standard skeleton of the Berkovich space $\mathbb{A}^n$ (over some fixed non-archimedean field k), let X be a k-affinoid space and let $f : X \to \mathbb{A}^n$ be an analytic morphism with zero-dimensional fibers. I will roughly present a proof of the fact that $f^{-1}(S_n)$ is piecewise monomial, i.e. locally looks like a compact subset of $(\mathbb{R}_+)^n$ defined by finitely many non-strict monomial inequalities. As a key tool it uses a compact, totally disconnected space of valuations associated to X and some of its tameness properties which themselves rely on slightly involved commutative algebra (excellent schemes, regular morphisms, unibranch schemes...).

This is joint work with Amaury Thuillier.

Thursday 10:00, Joe Rabinoff

The tropical skeleton

Given a closed subscheme X of a toric variety Y_Δ, we define a canonical locally closed subset $\mathrm{STrop}(X)$ of X^{an}, called the tropical skeleton, as the set of Shilov boundary points of the fibers of the tropicalization map. This is a kind of compactified c-skeleton in the sense of Ducros. Perhaps surprisingly, $\mathrm{STrop}(X)$ is not always closed; we will discuss equidimensionality conditions under which a limit of a sequence of points of $\mathrm{STrop}(X)$ is contained in $\mathrm{STrop}(X)$. We give applications to so-called schön subvarieties of a torus, in which case the tropical skeleton coincides with a canonically defined skeleton in the sense of Berkovich, as well as the parameterizing complex of Helm–Katz. We also give applications to continuity of the section of the tropicalization map on the tropical multiplicity-1 locus.

Thursday 11:00, Chengyang Xu

Skeleton and dual complex

In the first part of this talk, I will discuss a construction of the essential skeleton from the dual complex of a relative minimal model, for a variety X over $K = k((t))$ such that K_X is semi-ample. In the second part, I will discuss examples of the topology of the dual complex of special types of varieties.

Thursday 14:30, Michael Temkin

Wild coverings of Berkovich curves

I will describe the structure of finite morphisms between smooth Berkovich curves. The special accent will be on the description of the loci of points of multiplicity n and its relation to Herbrand function and the ramification theory. If time permits we will also talk about the different function associated to a morphism.

Thursday 16:00, Shou-Wu Zhang

A p-adic Waldspurger formula

In this talk, I will explain a p-adic Waldspurger formula proved by Bertolini–Darmon–Prasanna under the Heegner condition, and in full generality later by Liu–Zhang–Zhang. I will start with a classical Waldspurger formula on complex modular forms and a Gross–Zagier formula on rational modular forms, then define p-adic modular forms, p-adic L-functions, p-adic period integrals, and finally state a p-adic Waldspurger formula.

Thursday 17:00, Kentaro Mitsui

Closed points on torsors under abelian varieties

We show the existence of a separable closed point of small degree on any torsor under an abelian variety over a complete discrete valuation field under mild assumptions on the residue field of the valuation ring and the reduction of the abelian variety. To show the existence, we introduce and study minimal models of torsors under quasi-projective smooth group schemes.

Friday 08:30, Klaus Künnemann

Metrics and delta-forms in non-archimedean analytic geometry

We report on joint work with Walter Gubler from Regensburg. We consider metrics on line bundles over the non-archimedean analytification of an algebraic variety. Extending work by Chambert-Loir and Ducros we introduce delta-forms, discuss their basic properties, and describe their use in the construction of first Chern forms, Monge–Ampère measures, and local heights.

Friday 10:00, Omid Amini

Limit linear series and distribution of Weierstrass points

I will report on recent progress in constructing a general framework for the study of degenerations of linear series on degenerating families of smooth proper curves over a field of characteristic zero, using in particular tools from non-Archimedean geometry, generalizing the Eisenbud–Harris theory of limit linear series from the eighties to any semistable curve.I will then discuss an application to the problem of understanding the limiting behavior of Weierstrass points on such families. This leads to thefollowing non-Archimedean version of a theorem of Mumford and Neeman: let X be a smooth proper curve over a non-Archimedean field of residue characteristic zero, and $\mathscr{L}$ an ample line bundle on X. The Weierstrass points of powers of $\mathscr{L}$ are equidistributed according to the Zhang measure on the dual graph of a semistable model.

The talk is partially based on joint works with M. Baker and E. Esteves.

Friday 11:00, Kazuhiko Yamaki

Recent progress in the geometric Bogomolov conjecture

Let A be an abelian variety over a function field and let X be a closed subvariety of A. The geometric Bogomolov conjecture claims that if X has a dense subset of small points, then X should be a special subvariety, where a special subvariety is a sum of a torsion subvariety and a subvariety defined over the constant field. In a recent work, we prove that the above conjecture holds under the assumption of $\mathrm{codim}(X) = 1$ or $\dim(X) = 1$. In particular, we obtain the Bogomolov conjecture for curves in their Jacobian over function fields in full generality. In a very important step of the proof, the canonical measures over non-archimedean analytic spaces play crucial roles. We will explain how they are used.

Friday 14:30, Bertrand Rémy

Automorphisms of Drinfeld half-spaces over a finite fields

We show that the automorphism group of Drinfeld's half-space over a finite field is the projective linear group of the underlying vector space. The proof of this result uses analytic geometry in the sense of Berkovich over the finite field equipped with the trivial valuation. We also take into account extensions of the base field.

This is joint work with Amaury Thuillier and Annette Werner.

Friday 16:00, Vladimir Berkovich

Complex analytic vanishing cycles for formal schemes

Let K be a non-Archimedean field with nontrivial discrete valuation whose ring of integers K° contains $\mathbf{C}$ and $\mathbf{C} \xrightarrow{\sim} \widetilde{K}$, and let Π be the subgroup of Galois group G of K which is the preimage of $\mathbf{Z}$ under the canonical isomorphism $G \xrightarrow{\sim} \widehat{\mathbf{Z}}$. I will describe a construction that associates to every special formal scheme $\mathfrak{X}$ over K° an exact functor $\Lambda^{\cdot} \mapsto R\Psi_\eta^b(\Lambda^{\cdot}_{\mathfrak{X}_\eta})$ from the derived category of Π-modules to the derived category of abelian Π-sheaves on the complex analytification $\mathfrak{X}_s^b$ of the closed fiber $\mathfrak{X}_s$ of $\mathfrak{X}$. This functor possesses many of the properties established in my previous work for the similar functor on complexes of discrete *torsion G-modules* and extends the latter. Furthermore, given a morphism of complex analytic germs $(X, x) \to (\mathbf{C}, 0)$, a scheme $\mathscr{Y}$ of finite type over $\mathscr{O}_{X,x}$, a subscheme $\mathscr{Z} \subset \mathscr{Y} \otimes_{\mathscr{O}_{X,x}} \mathbf{C}$, and a homomorphism $\mathscr{O}_{\mathbf{C},0} \to K^\circ$ defined by a generator of the maximal ideal of K°, the above functor associated to the formal completion $\widehat{\mathscr{Y}}_{/\mathscr{Z}}$ coincides with the restriction of the corresponding complex analytic vanishing cycles functor (from SGA7), associated to the analytification $\mathscr{Y}^b$ of $\mathscr{Y}$, to $\mathscr{Z}^b$. The construction allows one to define, for every compact strictly K-analytic space X, integral "étale" cohomology groups $H^q(\overline{X}, \mathbf{Z})$ of $\overline{X} = X\widehat{\otimes}\widehat{K^a}$. These are finitely generated abelian groups provided with a quasi-unipotent action of Π and functorial in X, and they give rise to the étale l-adic cohomology groups of $\overline{X}$ and, if X is rig-smooth, to the de Rham cohomology groups of X.

Acknowledgments

We are grateful to the organizing committee, scientific committee, and our sponsors who dedicated time and resources to make this conference possible.

Organizing Committee

Gaetan Bisson	Université de la Polynésie française	French Polynesia
Jean Chaumine	Université de la Polynésie française	French Polynesia
Philippe Lebacque	Université de Besançon	France
Roger Oyono	Université de la Polynésie française	French Polynesia

Scientific Committee

Antoine Ducros	Université Paris VI	France
Johannes Nicaise	Katholieke Universiteit Leuven	Belgium
Jérôme Poineau	Université de Caen	France
Alexey Zykin	University of French Polynesia	French Polynesia

Sponsors

Agence Nationale de la Recherche
European Research Council
Université de la Polynésie française
Institut Universitaire de France

Participants

Alexey Zykin	Université de la Polynésie française	French Polynesia
Ambrus Pal	Imperial College London	United Kingdom
Antoine Ducros	Université Pierre et Marie Curie	France
Bertrand Rémy	Université Claude Bernard Lyon 1	France
Chengyang Xu	Beijing Center of Math. Research	China
Emmanuel Lepage	Université Pierre et Marie Curie	France
Éric Férard	Université de la Polynésie française	French Polynesia
Florent Martin	Universität Regensburg	Germany
Francesco Baldassarri	Università degli Studi di Padova	Italy
Fumiharu Kato	Kumamoto University	Japan
Gaëtan Bisson	Université de la Polynésie française	French Polynesia
Jean Chaumine	Université de la Polynésie française	French Polynesia
Jérôme Poineau	Université de Caen	France
Joe Rabinoff	Georgia Institute of Technology	United States
Juan Rivera-Letelier	Pontificia Universidad Católica	Chile
Kazuhiko Yamaki	Kyoto University	Japan
Kazuhiro Fujiwara	Nagoya University	Japan
Kentaro Mitsui	Kobe University	Japan
Klaus Künnemann	Universität Regensburg	Germany
Laura DeMarco	Northwestern University	United States
Marc Chapuis	Université Pierre et Marie Curie	France
Mattias Jonsson	University of Michigan	United States
Michael Temkin	Hebrew University of Jerusalem	Israel
Mihnea Popa	Northwestern University	United States
Omid Amini	École Normale Supérieure	France
Roger Oyono	Université de la Polynésie française	French Polynesia
Ruochuan Liu	Beijing Center of Math. Research	China
Shou-Wu Zhang	Princeton University	United States
Shu Kawaguchi	Doshisha University	Japan
Vladimir Berkovich	Weizmann Institute of Science	Israel
Ye Luo	Rice University	United States


Restaurants
Conference Hall
Boulevard de la Reine Pomare IV
Avenue du Général de Gaulle
Arrêt Nui Pouvana'a a 'Opa
Rue du Docteur Cassiau
Rue Dumont d'Ur
Rue du Chef Tenlerooterai
Rue Maudi Gobrait
Avenue du Commandant Destremau
Arrêt 5 no Mati 1797
Rue Vénus
Avenue des Poilus Tahitiens
Rue du Bataillon du Pacifique
Rue Cook
Rue Ch. Drollet
Les Jardins de Pā'ofa'i
Terrain de Beach Volley
Place Jacques Chirac
Musée de La Perle
Banque de Polynésie
Assemblée de Polynésie
HAUT-COMMISSARIAT RESIDENCE
IGAT
Vice Présidence
Institut De Recherche Médicale
Gendarmerie : COMGEND
Direction de la Police
MAISON DE LA CULTURE
Tahua To'ata
Paofai
Garde
College Tipaerui
Bambridge
Pk 1
TIKI PĒ'UE
LE RETRO
69 SLAM
TELECTRONIQUE
BUT
Les Petits Ventres
Socredo
Rt 6